

Homework September 4, 2019  
**Due: September 11.**

1. At the antiproton facility FAIR in Germany an antiproton beam will collide with either a hydrogen or deuterium target. (The target can be considered at rest in the lab system.)

a) What is the momentum (in the lab system) of the antiproton beam for producing the  $J/\psi$  resonance:  $\tilde{p} + p \rightarrow J/\psi$ ?

b) Consider a pentaquark baryon  $P_c$  that can decay as  $P_c \rightarrow J/\psi + n$ . At what mass of  $P_c$  the momentum necessary for its formation on the deuterium target,  $\tilde{p} + d \rightarrow P_c$ , is the same as the one found in the question a) ?

(Feel free to neglect small mass difference between proton and neutron as well as the binding energy of the neutron and proton in deuterium.)

(Look up the masses of the particles in the Tables.)

2. a) A charged pion  $\pi^+$  decays as  $\pi^+ \rightarrow \mu^+ \nu_\mu$ . What is the energy of the muon in the rest frame of the pion?

b) One of the decays of a charged Kaon is  $K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e$ . What is the maximal possible energy of the positron emerging from this decay (in the rest frame of the Kaon)?

3. The solar mass is about  $2 \times 10^{30}$  kg, the distance to the Sun is about 150 million kilometers and the angular size of the solar disk is about  $0.5^\circ$ . The energy flux arriving at the Earth as photons is about  $0.14 \text{ W/cm}^2$  (the so-called solar constant). The energy generation inside the sun is initiated by a slow (weak interaction) reaction  $p + p \rightarrow d + e^+ + \nu_e$ , where  $d$  is deuteron, followed by much faster electromagnetic and strong interaction processes (ending mostly in formation of  $^4\text{He}$ , and the total energy generated about 6 MeV per each initial proton), so that the overall probability is determined by the cross section of the slow initial reaction. Using these data estimate:

a) The cross section of the initial reaction (in  $\text{cm}^2$ )

b) The lifetime of the Sun (in years)

c) The total flux of neutrinos on Earth (in  $\text{cm}^{-2}\text{s}^{-1}$ ).

(You should be able to estimate the speed of the protons by considering the Sun as a gravitationally bound ball of gas.) This is an “order of magnitude estimate” type of problem. Thus moderate numerical factors, say, like 3, can be ignored.

**Due: September 18.**

1. Suppose that the Quantum Gravity (whatever that may be) does not conserve the baryon number and thus gives rise to proton decay. Using dimensional arguments, estimate, what lifetime of the proton one might then expect, if the only relevant parameters, determining the gravitational decay amplitude and the kinematical scale, are the Newton's constant  $G_N$  and the proton mass. Express your estimate for the lifetime in years.

2. In the decays of the muon,  $\mu \rightarrow e \tilde{\nu}_e \nu_\mu$ , and the tau lepton,  $\tau \rightarrow e \tilde{\nu}_e \nu_\tau$ , the mass of the electron can be neglected in comparison with the mass of the decaying lepton. Thus the only relevant parameters determining each decay rate are the Fermi constant,  $G_F$ , and the mass of the parent lepton: respectively  $m_\mu$  and  $m_\tau$ . Using dimensional arguments, find the ratio of the decay rates

$$\frac{\Gamma(\tau \rightarrow e \tilde{\nu}_e \nu_\tau)}{\Gamma(\mu \rightarrow e \tilde{\nu}_e \nu_\mu)}.$$

Compare your result with the data. (Tables! Or look up at <http://pdg.lbl.gov>) What accuracy of your prediction would you expect? How the expected accuracy compares with the experimental errors?

2. The  $\Delta^+$  resonance with the mass about 1230 MeV decays (dominantly) to  $N\pi$ , where  $N$  is a nucleon, or to  $p + \gamma$ . For this reason the cross section of the reaction  $\gamma + p \rightarrow \Delta^+ \rightarrow N\pi$  is large at the c.m. energy corresponding to the  $\Delta^+$  resonance and is about  $4 \times 10^{-28} \text{ cm}^2$  at the maximum. Consider this process for a very high energy proton moving through the Cosmic Microwave Background (CMB).

a) At what energy of the proton this process becomes significant (and thus leads to a degradation of the energy of the nucleon)?

b) What is the mean free path of the protons with such energy?

(This process sets an upper limit on the energy of cosmic ray protons that come from cosmological distances.)