Homework September 4, 2019

Due: September 11.

1. An the antiproton facility FAIR in Germany an antiproton beam will collide with either a hydrogen or deuterium target. (The target can be considered at rest in the lab system.)

a) What is the momentum (in the lab system) of the antiproton beam for producing the J/ψ resonance: $\tilde{p} + p \to J/\psi$?

b) Consider a pentaquark baryon P_c that can decay as $P_c \to J/\psi + n$. At what mass of P_c the momentum necessary for its formation on the deuterium target, $\tilde{p} + d \to P_c$, is the same as the one found in the question a) ?

(Feel free to neglect small mass difference between proton and netron as well as the binding energy of the neutron and proton in deuteron.)

(Look up the masses of the particles in the Tables.)

2. a) A charged pion π^+ decays as $\pi^+ \to \mu^+\nu_\mu$. What is the energy of the muon in the rest frame of the pion?

b) One of the decays of a charged Kaon is $K^+ \to \pi^+ \, \pi^- \, e^+ \, \nu_e$. What is the maximal possible energy of the positron emerging from this decay (in the rest frame of the Kaon)?

3. The solar mass is about 2×10^{30} kg, the distance to the Sun is about 150 million kilometers and the angular size of the solar disk is about 0.5°. The energy flux arriving at the Earth as photons is about 0.14 W/cm^2 (the so-called solar constant). The energy generation inside the sun is initiated by a slow (weak interaction) reaction $p+p \to d+e^+ +\nu_e$, where d is deuteron, followed by much faster electromagnetic and strong interaction processes (ending mostly in formation of ^{4}He, and the total energy generated about 6 MeV per each initial proton), so that the overall probability is determined by the cross section of the slow initial reaction. Using these data estimate:

a) The cross section of the initial reaction (in cm^2)

b) The lifetime of the Sun (in years)

c) The total flux of neutrinos on Earth (in cm^{-2}c^{-1}).

(You should be able to estimate the speed of the protons by considering the Sun as a gravitationally bound ball of gas.) This is an "order of magnitude estimate" type of problem. Thus moderate numerical factors, say, like 3, can be ignored.

Homework September 11, 2019

Due: September 18.

1. Suppose that the Quantum Gravity (whatever that may be) does not conserve the baryon number and thus gives rise to proton decay. Using dimensional arguments, estimate, what lifetime of the proton one might then expect, if the only relevant parameters, determining the gravitational decay amplitude and the kinematical scale, are the Newton's constant G_N and the proton mass. Express your estimate for the lifetime in years.

2. In the decays of the muon, $\mu \to e\,\tilde{\nu}_e\,\nu_\mu$, and the tau lepton, $\tau \to e\,\tilde{\nu}_e\,\nu_\tau$, the mass of the electron can be neglected in comparison with the mass of the decaying lepton. Thus the only relevant parameters determining each decay rate are the Fermi constant, G_F, and the mass of the parent lepton: respectively m_μ and m_τ. Using dimensional arguments, find the ratio of the decay rates

$$\frac{\Gamma(\tau \to e\,\tilde{\nu}_e\,\nu_\tau)}{\Gamma(\mu \to e\,\tilde{\nu}_e\,\nu_\mu)}\ .$$

Compare your result with the data. (Tables! Or look up at *http://pdg.lbl.gov*) What accuracy of your prediction would you expect? How the expected accuracy compares with the experimental errors?

2. The Δ^+ resonance with the mass about 1230 MeV decays (dominantly) to $N\pi$, where N is a nucleon, or to $p+\gamma$. For this reason the cross section of the reaction $\gamma + p \to \Delta^+ \to N\pi$ is large at the c.m. energy corresponding to the Δ^+ resonance and is about $4 \times 10^{-28}\,\mathrm{cm}^2$ at the maximum. Consider this process for a very high energy proton moving through the Cosmic Microwave Background (CMB).
a) At what energy of the proton this process becomes significant (and thus leads to a degradation of the energy of the nucleon)?
b) What is the mean free path of the protons with such energy?
(This process sets an upper limit on the energy of cosmic ray protons that come from cosmological distances.)

Homework #3, September 18, 2019

Due: September 25.

1. A supernova type II explosion is a spectacular process in which a stellar mass approximately equal to the mass of the Sun (2×10^{33} g) collapses to a radius of approximately 10 km. The difference in the gravitational binding energy is emitted dominantly as neutrinos with typical energy 10 MeV.

The supernova SN1987a, observed in February 1987, had exploded at the distance of about 170 thousand light years from the solar system.

- Estimate the flux of neutrinos from SN1987a at the Earth.
- Using dimensional considerations, estimate the cross-section for neutrino-electron scattering, and find the number of the scattering events expected in a 100 metric ton detector.
- The emission of neutrinos lasts few seconds (take it as 10 s for an estimate). If events with 10 MeV neutrinos and 30 MeV neutrinos from the SN1987a, were observed within 10 seconds from each other, what limit on the neutrino mass can be inferred?

2. Consider the β decay ${}^3\text{H} \to {}^3\text{He} + e + \tilde{\nu}_e$. Assume that the initial state is a tritium atom in the ground state, i.e. there is a "spectator" electron. The electric charge of the nucleus, that binds the "spectator" electron, clearly changes as a result of the decay. Thus the spectator electron in the final state can end up in various energy states of a helium ion. Find (the formula and the numbers in %) the probability of the final ion to be in:
i) the ground state,
ii) the first excited state.

3. If you know the lifetime of neutron, can you approximately estimate the rate of the β decay of the charged pion: $\pi^- \to \pi^0 e^- \bar{\nu}_e$? Compare your prediction with the data.