

Finite-Temperature Field Theory

Principles and Applications

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Preface

What happens when ordinary matter is so greatly compressed that the electrons form a relativistic degenerate gas, as in a white dwarf star? What happens when the matter is compressed even further so that atomic nuclei overlap to form superdense nuclear matter, as in a neutron star? What happens when nuclear matter is heated to such great temperatures that the nucleons and pions melt into quarks and gluons, as in high-energy nuclear collisions? What happened in the spontaneous symmetry breaking of the unified theory of the weak and electromagnetic interactions during the big bang? Questions like these have fascinated us for a long time. The purpose of this book is to develop the fundamental principles and mathematical techniques that enable the formulation of answers to these mind-boggling questions. The study of matter under extreme conditions has blossomed into a field of intense interdisciplinary activity and global extent. The analysis of the collective behavior of interacting relativistic systems spans a rich palette of physical phenomena. One of the ultimate goals of the whole program is to map out the phase diagram of the standard model and its extensions.

This text assumes that the reader has completed graduate level courses in thermal and statistical physics and in relativistic quantum field theory. Our aims are to convey a coherent picture of the field and to prepare the reader to read and understand the original and current literature. The book is not, however, a compendium of all known results; this would have made it prohibitively long. We start from the basic principles of quantum field theory, thermodynamics, and statistical mechanics. This development is most elegantly accomplished by means of Feynman's functional integral formalism. Having a functional integral expression for the partition function allows a straightforward derivation of diagrammatic rules for interacting field theories. It also provides a framework for defining gauge theories on finite lattices, which then enables integration by Monte Carlo

techniques. The formal aspects are illustrated with applications drawn from fields of research that are close to the authors' own experience. Each chapter carries its own exercises, reference list, and select bibliography.

The book is based on *Finite-Temperature Field Theory*, written by one of us (JK) and published in 1989. Although the fundamental principles have not changed, there have been many important developments since then, necessitating a new book.

We would like to acknowledge the assistance of Frithjof Karsch and Steven Gottlieb in transmitting some of their results of lattice computations, presented in Chapter 10, and Andrew Steiner for performing the numerical calculations used to prepare many of the figures in Chapter 11. We are grateful to a number of friends, colleagues, and students for their helpful comments and suggestions and for their careful reading of the manuscript, especially Peter Arnold, Eric Braaten, Paul Ellis, Philippe de Forcrand, Bengt Friman, Edmond Iancu, Sangyong Jeon, Keijo Kajantie, Frithjof Karsch, Mikko Laine, Stefan Leupold, Guy Moore, Ulrich Mosel, Robert Pisarski, Brian Serot, Andrew Steiner, and Laurence Yaffe.