Coulomb Drag in Quantum Circuits

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We study the drag effect in a system of two electrically isolated quantum point contacts, coupled by Coulomb interactions. Drag current exhibits maxima as a function of quantum point contacts gate voltages when the latter are tuned to the transitions between quantized conductance plateaus. In the linear regime this behavior is due to enhanced electron-hole asymmetry near an opening of a new conductance channel. In the nonlinear regime the drag current is proportional to the shot noise of the driving circuit, suggesting that the Coulomb drag experiments may be a convenient way to measure the quantum shot noise. Remarkably, the transition to the nonlinear regime may occur at driving voltages substantially smaller than the temperature.

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The drag effect in bulk 2D systems is well established experimentally [1–6] and studied theoretically [7–10]. By now it is one of the standard ways to access and measure electron-electron scattering. Very recently a number of experiments were performed to study Coulomb drag in quantum confined geometries such as quantum wires [11–14], quantum dots [15,16], or quantum point contacts (QPC) [17]. In these systems a source-drain voltage V is applied to generate current in the *drive circuit* while an induced current (or voltage) is measured in the *drag circuit*. Such a drag current is a function of the drive voltage V as well as gate voltages, which control transmission of one or both circuits. Figure 1(a) shows an example of such a setup, where both drive and drag circuits are represented by two QPC's.

It was reported [11–13,16,17] that the drag current exhibits maxima for specific values of the gate voltage, where the drive QPC is tuned to an opening of another conductance channel. This observation is depicted schematically in Fig. 1(b). It was attributed to the shot noise of the drive QPC [15–18], which is known [19,20] to exhibit a qualitatively similar behavior. The idea is that the drag circuit serves as a detector and a rectifier of the *quantum* shot noise in the drive circuit. Although plausible and in a certain regime indeed correct, this mechanism differs substantially from the one familiar from the bulk 2D drag effect. In the latter case drag may be interpreted [9] as a rectification of nearly equilibrium *classical* thermal fluctuations in the drive circuit. As a result the drag current is a power-law function of the temperature ($\sim T^2$ in many cases [21]). Such a rectification is only possible due to electron-hole asymmetry in both circuits (otherwise drag currents of electrons and holes cancel each other). In the bulk systems the asymmetry is due to a small curvature of the particle dispersion relation near the Fermi energy.

Mesoscopic and quantum circuits with the spatial dimensions less than the temperature length $L_T = v_F/T$ and voltage length $L_V = v_F/eV$ differ from the bulk 2D systems in several important ways. (i) The electron-hole symmetry in such devices is broken much more strongly than in bulk systems. In mesoscopic devices this is due to a random configurations of impurities [23], while in the QPC's the effect is due to the energy dependence of transmission coefficients. Because of the latter the hole's transmission probability is typically less than that of the electrons. (ii) The spatial inversion symmetry may be broken by left-right asymmetry of the circuit design. As we show below, this makes two polarities of the drive voltage V to be essentially nonequivalent. (iii) Because of the above, the quantum circuits may be easily driven out of the linear

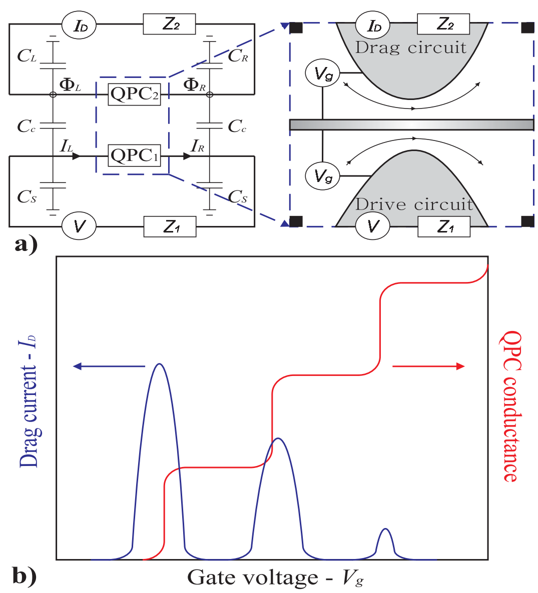

FIG. 1 (color online). (a) Two coupled QPCs and surrounding electric circuitry. The Coulomb coupling is due to mutual capacitances C_c. Gate voltage V_g control transmission of, e.g., drive QPC. (b) Schematic representation of linear conductance of the drive QPC along with the drag current as a function of the gate voltage.

response domain (unlike the bulk systems). Typically a voltage needed to drive a quantum circuit into a nonlinear regime is parametrically smaller than the temperature.

In this Letter we study the drag effect between two QPC's, Fig. 1(a). We assume weak interaction between the two circuits mediated by mutual capacitances C_c, Fig. 1(a). Since the external circuits typically also include dissipative elements, the actual interaction is, in general, frequency dependent [15] and determined by a *matrix* of trans-impedances (see below). Because of the weak coupling the drag current I_D is small, and therefore, the drag circuit is assumed to be close to equilibrium. On the other hand, the drive circuit may be substantially out of equilibrium, due to an applied bias V. With these assumption we evaluate the drag current I_D in the second-order in the intercircuit interactions in the framework of the Keldysh diagrammatic technique (to account for nonequilibrium conditions of the drive circuit). The details of the calculations are reported in Ref. [24].

We show that at sufficiently small driving voltage V the drag current is linear, $I_D \propto V$. In this regime the mechanism of the drag is similar to that in the bulk 2D systems: i.e., rectification of near-equilibrium thermal noise. Consequently $I_D \propto T^2$ at small temperatures. The rectification relies on the electron-hole asymmetry, which is due to energy dependence of the transmission probability in a given channel. The asymmetry is the strongest near an opening of a new conductance channel. Indeed, in this case thermally excited electrons are much more likely to be transmitted than the holes. Hence the behavior sketched in Fig. 1(b) (though with no relation to the quantum shot noise). At larger drive voltages $I_D \propto V^2$, and the effect is indeed due to the detection of the *excess* shot noise of the drive circuit [15–18]. The energy dependence of the transmission probability is not required in this regime, and I_D is proportional to the celebrated Fano factor [19,20,25]. Remarkably, the crossover between the two regimes takes place at $eV \sim T^2/\Delta \ll T$, where Δ is an energy scale of the transmission probability.

Quantitatively we found the following expression for the drag current:

$$I_D(V) = \int \frac{d\omega}{4\pi\omega^2} \mathrm{Tr}[\hat{\mathcal{Z}}(\omega)\hat{S}_1(\omega, V)\hat{\mathcal{Z}}(-\omega)\hat{\Gamma}_2(\omega)]. \quad (1)$$

Here and throughout the Letter indices 1, 2 refer to the drive and drag circuits, correspondingly. The elements $\mathcal{Z}_{ab}(\omega)$, $a, b = R, L$ of the trans-impedance matrix $\hat{\mathcal{Z}}(\omega)$ encode intercircuit coupling. They are defined as $\mathcal{Z}_{ab}(\omega) = \partial\Phi_a(\omega)/\partial I_b(\omega)$, where the corresponding local fluctuating currents I_a and voltages Φ_a are indicated in Fig. 1(a).

In Eq. (1) the drive circuit is characterized by the *excess* part $S_1^{ab}(\omega, V) = S_{ab}(\omega, V) - S_{ab}(\omega, 0)$ of current-current correlation matrix $S_{ab}(\omega, V) = \int dt e^{i\omega t}\langle\langle \delta\hat{I}_a(t)\delta\hat{I}_b(0) + \delta\hat{I}_b(0)\delta\hat{I}_a(t)\rangle\rangle$, which is known from the theory of quantum shot noise [19,26,27]. In particular

$$S_{LL}(\omega, V) = \frac{2}{R_Q}\sum_n \int d\epsilon [B_{RR}(\epsilon)|\mathbf{t}_n^R(\epsilon_+)|^2|\mathbf{t}_n^R(\epsilon_-)|^2 + B_{LL}(\epsilon)[1 - \mathbf{r}_n^{*L}(\epsilon_+)\mathbf{r}_n^L(\epsilon_-)] \times [1 - \mathbf{r}_n^{*L}(\epsilon_-)\mathbf{r}_n^L(\epsilon_+)] + B_{LR}(\epsilon)|\mathbf{r}_n^L(\epsilon_+)|^2 \times |\mathbf{t}_n^R(\epsilon_-)|^2 + B_{RL}(\epsilon)|\mathbf{t}_n^R(\epsilon_+)|^2|\mathbf{r}_n^L(\epsilon_-)|^2], \quad (2)$$

with similar expressions for the S_{LR}, S_{RL}, and S_{RR} components; see Refs. [24,26] for details. Here $R_Q = \frac{2\pi\hbar}{e^2}$ is quantum resistance; $|\mathbf{t}_n^{L(R)}(\epsilon)|^2 = |\mathbf{t}_n(\epsilon + eV_{L(R)})|^2$ are transmission probabilities of the drive QPC_1, labeled by the transverse channel index n; $|\mathbf{r}_n^{L(R)}(\epsilon)|^2 = 1 - |\mathbf{t}_n^{L(R)}(\epsilon)|^2$ and $V_L - V_R = V$. The statistical factors are $B_{ab}(\epsilon) = f_a(\epsilon_+)[1 - f_b(\epsilon_-)] + f_b(\epsilon_-)[1 - f_a(\epsilon_+)]$, with $f_{L(R)}(\epsilon) = f(\epsilon + eV_{L(R)})$ being the Fermi distributions of the two leads and $\epsilon_\pm = \epsilon \pm \omega/2$.

The drag circuit in Eq. (1) is characterized by the rectification coefficient $\hat{\Gamma}_2(\omega) = \Gamma_2(\omega)\hat{\tau}_z$ of the ac voltage fluctuations applied to the (near-equilibrium) drag QPC_2, where $\hat{\tau}_z$ is third Pauli matrix acting in the left-right space. Rectification is given by

$$\Gamma_2(\omega) = \frac{2e}{R_Q}\sum_n \int d\epsilon [f(\epsilon_-) - f(\epsilon_+)][|\mathbf{t}_n(\epsilon_+)|^2 - |\mathbf{t}_n(\epsilon_-)|^2]. \quad (3)$$

Characteristics of the QPC_2 enter through its energy-dependent transmission probabilities $|\mathbf{t}_n(\epsilon)|^2$. This expression admits a transparent interpretation: potential fluctuations with frequency ω, say on the left of the QPC, create electron-hole pairs with energies $\epsilon_\pm$ on the branch of right moving particles. Consequently the electrons can pass through the QPC with the probability $|\mathbf{t}_n(\epsilon_+)|^2$, while the holes with the probability $|\mathbf{t}_n(\epsilon_-)|^2$. The difference between the two gives the dc current flowing across the QPC. Notice that the energy dependence of the transmission probabilities in the drag QPC is crucial [28] in order to have the asymmetry between electrons and holes, and thus nonzero rectification $\Gamma_2(\omega)$.

Focusing on a single partially open channel in a smooth QPC, one may think of the potential barrier across it as being practically parabolic. In such a case its transmission probability is given by

$$|\mathbf{t}(\epsilon)|^2 = [\exp\{(eV_g - \epsilon)/\Delta_2\} + 1]^{-1}, \quad (4)$$

where Δ_2 is an energy scale associated with the curvature of the parabolic barrier in the QPC_2 and gate voltage V_g moves the top of the barrier relative to the Fermi energy. This form of transmission was used to explain QPC conductance quantization [29] and it turns out to be useful in application to the Coulomb drag problem. Inserting (4) into (3) and carrying out energy integration, one finds

$$\Gamma_2(\omega) = \frac{2e\Delta_2}{R_Q} \ln\left(1 + \frac{\sinh^2(\omega/2\Delta_2)}{\cosh^2(eV_g/2\Delta_2)}\right) \quad (5)$$

for $T \ll \Delta_2$. In the other limit, $T \gg \Delta_2$, one should replace $\Delta_2 \to T$ in the Eq. (5). Notice that for small frequency $\omega \ll \Delta_2$ one has $\Gamma_2 \sim \omega^2$, making the integral in Eq. (1) to be convergent in $\omega \to 0$ region.

Linear drag regime.—For small applied voltages V one expects the response current I_D to be linear in V. Expanding $\hat{S}_1(\omega, V)$ to the linear order in V, one finds that only diagonal components of the current-current correlation matrix contribute to the linear response and as a result

$$\hat{S}_1(\omega, V) = V \frac{\partial}{\partial \omega}\left[\coth\frac{\omega}{2T}\right]\hat{\Gamma}_1(\omega) + O(V^3), \tag{6}$$

where $\hat{\Gamma}_1(\omega) = \Gamma_1(\omega)\hat{\tau}_z$ and $\Gamma_1(\omega)$ is obtained from Eq. (3) by substituting transmission probabilities of QPC_2, by that of QPC_1. Inserting Eq. (6) into Eq. (1) one finds

$$I_D = V\frac{R_Q^2}{4\pi}\int d\omega \frac{\alpha_+(\omega)}{\omega^2}\frac{\partial}{\partial \omega}\left[\coth\frac{\omega}{2T}\right]\Gamma_1(\omega)\Gamma_2(\omega), \tag{7}$$

where the dimensionless interaction kernel $\alpha_+(\omega)$ is expressed through the components of the trans-impedance matrix as $\alpha_\pm(\omega) = [(|Z_{LL}|^2 - Z_{LR}Z_{RL}) \pm (|Z_{RR}|^2 - Z_{LR}Z_{RL})]/2R_Q^2$. Derived Eq. (7) has the same general structure as the one for the drag current in bulk 2D systems [9,10]. Being symmetric with respect to $1 \leftrightarrow 2$ permutation, it satisfies the Onsager relation for the linear response coefficient.

Assuming the load impedance of the drag circuit to be much larger than that of the drive one $Z_1 \ll Z_2 \ll R_Q$ and the mutual capacitance of the two circuits to be small $C_c \ll C_{R,L,s}$, see Fig. 1(a), one finds for the low frequency limit $\omega \ll (Z_1C_s)^{-1}$ of the interaction kernels

$$\alpha_\pm(0) = \frac{Z_1^2}{8R_Q^2}\frac{C_c^2}{C_L^2C_R^2}\begin{cases}2C_L^2 + 2C_LC_R + 2C_R^2 \\ C_L^2 - C_R^2\end{cases}. \tag{8}$$

For $Z_1 \to 0$ the drive QPC is shorted and the drag circuit is insensitive to the fluctuations. Substituting now Eq. (5) into Eq. (7), one finds for, e.g., low-temperature regime $T \ll \Delta_{1,2}$

$$I_D = \frac{V}{R_Q}\frac{\alpha_+(0)\pi^2}{6}\frac{T^2}{\Delta_1\Delta_2}\frac{1}{\cosh^2(eV_g/2\Delta_1)}, \tag{9}$$

where we assumed that the gate voltage of QPC_2 is tuned to adjust the top of its barrier with the Fermi energy and wrote I_D as a function of the gate voltage in QPC_1. We have also assumed that $T \ll (Z_1C_s)^{-1}$ to substitute $\alpha_+(\omega)$ by its dc limit Eq. (8). The resulting expression exhibits a peak at $V_g = 0$ similar to that depicted in Fig. 1(b). Yet it has nothing to do with the shot noise, but rather reflects rectification of near-equilibrium thermal fluctuations (hence the factor T^2) along with the electron-hole asymmetry (hence a nonmonotonic dependence on V_g). For monotonically increasing functions $|\mathbf{t}(\epsilon)|^2$ in both circuits the linear drag is positive (i.e., currents flow in the same direction).

Nonlinear regime.—At larger drive voltages drag current ceases to be linear in V. Furthermore, contrary to the linear response case, $\hat{S}_1(\omega, V)$ does not require energy dependence of the transmission probabilities and could be evaluated for energy independent $|\mathbf{t}_n|^2$ (this is a fair assumption for $T, eV \ll \Delta_1$). Assuming in addition $T \ll eV$, one finds a celebrated expression for the quantum shot noise [19,27]

$$\hat{S}_1(\omega, V) = 2\frac{|eV + \omega| + |eV - \omega|}{R_Q}\sum_n |\mathbf{t}_n|^2[1 - |\mathbf{t}_n|^2]\hat{\tau}_0. \tag{10}$$

Inserting Eq. (10) into Eq. (1), after frequency integration bounded by the voltage, one finds for the drag current [30]

$$I_D = \frac{eV^2}{\Delta_2 R_Q}\alpha_-(0)\sum_n |\mathbf{t}_n|^2[1 - |\mathbf{t}_n|^2]. \tag{11}$$

Here again we assumed that the detector QPC_1 is tuned to the transition between the plateaus. We also assumed $eV \ll (Z_1C_s)^{-1}$ to substitute $\alpha_-(\omega)$ by its dc value, Eq. (8). One should notice that while $\alpha_+ > 0$, the sign of α_- is arbitrary. For a completely symmetric circuit $\alpha_- = 0$, while for an extremely asymmetric one $|\alpha_-| \lesssim \alpha_+/2$. Although we presented the derivation of Eq. (11) for $T \ll eV$, one may show that it remains valid at any temperature as long as $T \ll \min\{\Delta_1, (Z_1C_s)^{-1}\}$.

Equation (11) indeed shows that the drag current is due to the rectification of the quantum shot noise and hence proportional to the Fano factor [19]. It again exhibits a generic behavior depicted in Fig. 1(b), but the reason is rather different from the similar behavior in the linear regime. The direction of the nonlinear drag current is determined by the inversion asymmetry of the circuit (through the sign of α_-) rather than the direction of the drive current. As a result, for a certain polarity of the drive voltage, the drag current appears to be *negative*.

We discuss now a crossover between the two regimes. Assuming that for a generic circuit $\alpha_+ \sim \alpha_-$ and comparing Eqs. (9) and (11) one concludes that the transition from the linear to the nonlinear regime takes place at $V \approx V^*$ with

$$eV^* = T^2/\Delta_1 \ll T, \tag{12}$$

for $T \ll \Delta_1$. In the opposite limit, $T > \Delta_1$, the crossover voltage is given by the temperature $eV^* = T$. However, for a circuit with an almost perfect inversion symmetry, i.e., $\alpha_- \ll \alpha_+$, the nonlinear regime may be pushed to substantially larger voltages. Such a symmetric circuitry is not well suited for detection of the quantum shot noise.

Mesoscopic circuits.—One or both circuits may be represented by a multichannel quasi-1D (or 2D) mesoscopic sample. In this case $\sum_n |\mathbf{t}_n(\epsilon)|^2 = g(\epsilon)$ is a dimensionless (in units of R_Q^{-1}) conductance of the sample as a function of its Fermi energy. Such a conductance exhibits universal conductance fluctuations [31], that is $g(\epsilon) = g + \delta g(\epsilon)$, where $g \gg 1$ is an average conductance and $\delta g(\epsilon) \sim 1$ is a sample and energy-dependent fluctuating part. The characteristic scale of the energy dependence of the fluctuating part is the Thouless energy $E_{\rm Th} = \hbar D/L^2$, where D is the electronic diffusion constant and L is the sample size. Employing Eq. (3), one finds that the rectification coefficient of a given mesoscopic sample may be estimated as

$$\Gamma(\omega) \sim \pm \frac{e}{R_Q}\frac{\omega^2}{E_{\rm Th}}, \qquad \{T, \omega\} \ll E_{\rm Th}. \tag{13}$$

On the other hand, the nonequilibrium part of the noise correlator Eq. (10) exhibits a well-defined average value

$$S_1(\omega, V) = 2(|eV + \omega| + |eV - \omega|)\frac{g}{3R_Q}, \tag{14}$$

the coefficient $1/3$ is specific to a quasi-1D geometry [25].

In the Coulomb drag setup, where both circuits are represented by mesoscopic elements, employing Eqs. (1) and (7) along with (13) and (14), one finds for the drag current (both linear and nonlinear)

$$I_D \sim \frac{V}{R_Q}\left(\alpha_+ \frac{T^2}{E_{\rm Th}^2} + \alpha_- \frac{eV}{E_{\rm Th}} g\right), \tag{15}$$

where $T < E_{\rm Th}$. If the load impedance of the drive circuit is $Z_1 \sim g^{-1}$, then the linear in V term of Eq. (15) is in agreement with the corresponding result of Ref. [23]. The crossover between linear and nonlinear regimes takes place at $eV^* = \alpha_+ T^2/(\alpha_- g E_{\rm Th})$ which may be much less than both T and $E_{\rm Th}$. As a result, one may expect drag current to be substantially bigger than the linear response prediction already at the very modest bias voltage.

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$$\delta I_D(V) = \int \frac{d\omega}{4\pi}\frac{Z_{LL} - Z_{RR}}{\omega^2}[Z_{RL}\delta S_1^{LR} + Z_{LR}\delta S_1^{RL}]\Gamma_2(\omega).$$

It does not contribute either to the linear or to the nonlinear response regimes discussed in the text. Because of the symmetry of δS_1^{LR} and δS_1^{RL} with respect to the change $V \to -V$, at small voltages $\delta I_D \propto V^2$ in contrast to $I_D \propto V$ [Eq. (9)]. For the nonlinear regime, when energy dependence of the transmissions can be neglected, $\delta S_1^{LR} = \delta S_1^{RL}$, and δI_D vanishes, since $Z_{LR} = -Z_{RL}$.
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