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Coherent vs incoherent pairing in cuprates.

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We study the superconducting instability for cuprates within the spin-fermion model with hot spots. We show that the pairing problem is universal and involves fermions only near hot spots. We derived and solved the Eliashberg-type equations for the pairing instability temperature T_{ins}. We found that T_{ins} increases with underdoping and saturates at $\xi = \infty$. We argue, however, that the pairing problem is dominated by incoherent fermionic excitations, and T_{ins} is the onset of the pseudogap behavior rather than the actual T_c which in our theory vanishes at $\xi = \infty$. It is shown that the pseudogap has $d_{x^2-y^2}$ symmetry, but is mostly concentrated near hot spots.

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The unusual features of cuprates continue to attract a lot of attention motivated by a search for a new physics in strongly correlated fermionic systems. One of these features is a large difference observed in underdoped cuprates between the temperature T^* when the system begins displaying anomalous properties, and T_c, the actual temperature of the superconducting transition. The region between T^* and T_c is called the pseudogap phase. With underdoping, the pseudogap region widens as T_c goes down while T^* increrases.

In this communication, we reconsider the computations of the pairing instability temperature in the spin-fermion model for cuprates, and show that these computations provide a clue to the existence of the pseudogap phase.We show that for spin-mediated interaction, the d-wave pairing problem for sufficiently large magnetic correlation length ξ is self-consistently confined to fermionic momenta near hot spots (points at the Fermi surface separated by the antiferromagnetic momentum Q). We obtain an expression for the pairing instability temperature T_{ins} in the Eliashberg formalism. We show that at weak/moderate couplings, which we attribute to overdoped cuprates, the pairing problem is almost identical to that for phonon-mediated superconductors, the role of the Debye frequency is played by the spin fluctuation frequency ω_{sf}.

We however argue that in the strong coupling regime, which we associate with underdoped cuprates, the physics of the pairing instability is qualitatively different from that at weak/moderate couplings. We show that at strong coupling, the pairing is dominated by fermions with energies larger than ω_{sf}, which give rise to the pairing instability at $T_{ins} \gg \omega_{sf}$. These fermions display a non-Fermi liquid fully incoherent quantum-critical behavior. We conject that in this situation, the pairing instability corresponds to the onset of the pseudogap behavior (i.e., $T_{ins} = T^*$), while the actual superconductivity requires fermionic coherence and appears only at a much lower $T_c \propto \omega_{sf}$. We also show that the pseudogap below T_{ins} has $d_{x^2-y^2}$ symmetry, but more rapidly decays away from hot regions than $\cos k_x - \cos k_y$ form.

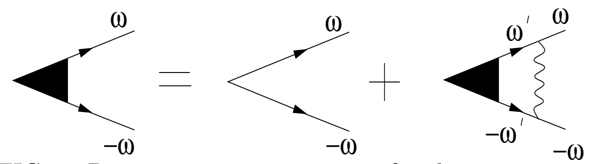

FIG. 1. Diagramatic representation for the pairing vertex. The solid and wavy lines are fermionic and spin fluctuation propagators, respectively.

The calculations of T_c and the angular dependence of the superconducting gap for spin-mediated pairing have been previously performed by various groups [1–6]. We compare our and earlier results later in the paper, after we describe our calculations.

A convenient way to study the pairing problem is to analyse a response to an external anomalous particle-particle vertex. The divergence of the response is the indication of a pairing instability. For this purpose we introduce $F^{(0)}-$ an infinitesimally small anomalous vertex with zero total momentum and frequency, and consider the equation for the fully renormalized vertex F^{full}. In the Eliashberg formalism, which accuracy we discuss below, the fully renormalized vertex is obtained by summing up the ladder series of diagrams and is given by

$$F^{full}_k(\omega_m) = F^{(0)}_k(\omega_m) - T\sum_{\omega'}\int \frac{d^2k'}{(2\pi)^2} F^{full}_{k'}(\omega') \times G_{k'}(\omega')\ G_{-k'}(-\omega')\ \Gamma(k-k', \omega-\omega'), \quad (1)$$

where $G_k(\omega)$ is the fully renormalized normal state single-particle Green's function and Γ is the pairing interaction. Both F^{full} and $F^{(0)}$ generally depend on relative fermionic momentum k and Matsubara frequency ω. At $T = T_{ins}$, Eq.(1) possesses a nontrivial solution for $F^{full}(k,\omega)$ even when $F^{(0)}$ vanishes.

Eq.(1) is an integral equation in both momentum and frequency. Usually the momentum integration can be simplified by exploring the spatial symmetry of the problem. In cuprates this symmetry is tetragonal up to minor orthorhombic distortions, which we neglect. The corresponding space group D_4 contains four $1D$ irreducible representations $A_{1g}, A_{2g}, B_{1g}, B_{2g}$, and one $2D$ irreducible representation E [7]. The eigenfunctions from different representations decouple completely in the pairing channel [8]. However, each representation contains an infinite number of eigenfunctions which do not decouple. This obviously complicates the integration over momentum.

By all experimental accounts, the pairing gap in cuprates is a spin singlet and has a B_{1g} ($d_{x^2-y^2}$) symmetry. The corresponding eigenfunctions are $d_m(\mathbf{k}) = \cos mk_x - \cos mk_y$, where m runs from 1 to infinity. Earlier calculations of T_c [1–5] were performed by expanding the vertex functions and the magnetically mediated pairing interaction $\Gamma(\mathbf{k}-\mathbf{k}', \omega-\omega') = 3\, g^2\, \chi(\mathbf{k}-\mathbf{k}', \omega-\omega')$, as $F_{B_{1g}} = \sum F_n(\omega) d_n(\mathbf{k})$, $\Gamma_{B_{1g}} = \sum \Gamma_n(\omega-\omega') d_n(\mathbf{k}) d_n(\mathbf{k}')$, where g is the spin-fermion coupling constant, and $\chi(q, \Omega)$ is the dynamical spin susceptibility. One then had to solve numerically an infinite set of coupled equations for F_n. For large magnetic correlation length ξ, this procedure is rather involved as $\chi(q, 0) = \chi^0/(\xi^{-2} + (q-Q)^2)$, and all Γ_n with $n \leq \xi$ are almost identical.

We, however, will demonstrate below that at large ξ the momentum integration in Eq. (1) is confined to the regions around hot spots. In this situation, fermionic states far away from k_{hs} are irrelevant for pairing, and it is better to evaluate the momentum integral directly, without expanding in $d_n(k)$.

To integrate over momenta in Eq.(1) we need the forms of the fully renormalized single-particle Green's function and the dynamical spin susceptibility in the normal state. The earlier studies of the spin-fermion model [9,10] have shown that both $G_k(\omega)$ and $\chi(q, \Omega)$ are determined self-consistently in terms of three input parameters: the effective spin-fermion coupling $\bar{g} = g^2\chi_0$, the Fermi velocity v_F, and the spin correlation length ξ. The dimensionless ratio of these parameters $\lambda = 3\bar{g}/(4\pi v_F \xi^{-1})$ measures the relative strength of the spin fermion scattering [9]. NMR and ARPES data indicate that $\lambda \geq 1$ at optimal doping, and increases with underdoping [1,3,9]. In this limit, a conventional perturbative expansion does not converge, but one can rearrange perturbation series and construct a new vacuum state which self-consistently includes the leading ($\sim \lambda$) bosonic and fermionic self-energies [9,10]. The remaining self-energy and vertex corrections scale as powers of $(1/N)\log\lambda$, where $N = 8$ is the number of hot spots in the Brillouin zone, and can be formally studied in RG formalism by treating N as a large number. In practice, however, $N = \infty$ theory is sufficient as the $1/N$ corrections have small prefactors [11] and are relevant only in a tiny region near $\xi = \infty$.

Near hot spots the $N = \infty$ calculations yield in Matsubara frequencies $G_k^{-1}(\omega_m) = i\omega_m Z_k(\omega_m) - \mathbf{v}_F(\mathbf{k}-\mathbf{k}_{hs})$ and $\chi(q, \Omega_m) = \chi_0\xi^2/(1 + (q-Q)^2\xi^2 + \Pi_Q(\Omega_m))$, where

$$Z_k(\omega_m) = 1 + \frac{2\ \lambda\ \Psi_k}{1 + \sqrt{1 + \frac{|\omega_m|}{\omega_{sf}}}};\ \Pi_Q(\Omega_m) = \frac{|\Omega_m|}{\omega_{sf}} \quad (2)$$

and $\omega_{sf} = (3/(2N))\ (v_F\xi^{-1})/\lambda \propto \xi^{-2}$ [9]. The function Ψ_k depends on $x = (\mathbf{k} - \mathbf{k}_{hs})^2\xi^2/(1 + |\omega_m|/\omega_{sf})$ and accounts for the decrease of the self-energy at deviations from hot spots ($\Psi(x = 0) = 1$, and $\Psi(x \gg 1) \sim x^{-1/2}$).

We see from Eq.(2) that for $\omega \leq \omega_{sf}$, $G_{k_F}^{-1}(\omega_m) \approx iZ(0)\omega_m$, i.e., as long as ξ is finite, the system preserves the Fermi-liquid behavior at the lowest frequencies. At larger frequencies $\omega \geq \omega_{sf}$, the system crosses over to a region, which is in the basin of attraction of the quantum critical point $\xi = \infty$. In this region $G_{k_F}^{-1}(\omega_m) \approx (3i/\sqrt{2\pi})\ |\bar{g}\omega/N|^{1/2}\text{sgn}(\omega)$ [3,9].

We now substitute the single-particle Green's function and the spin susceptibility into Eq.(1). Estimating the $2D$ momentum integral over k', we find that typical momenta along the Fermi surface ($k' = k_\parallel$) are of order $\xi^{-1}(1 + |\omega_m|/\omega_{sf})^{1/2}$, and the typical momenta in the direction along the Fermi velocity at a hot spot ($k' = k_\perp$) are at least by a factor $1/N$ smaller. Below we demonstrate that typical frequencies ω_m for the pairing problem are of order $\omega_{sf}\lambda^2$. Then typical $k_\parallel$ are of order ξ^{-1} for $\lambda = O(1)$ and $\bar{g}/v_F k_F$ for $\lambda \gg 1$. Our analysis, based on the expansion near hot spots, is therefore valid for all $\lambda \geq 1$ if $\bar{g}/v_F k_F \leq 1$, which we assume to hold.

Because typical $k_\perp \ll k_\parallel$, the integration over $k_\perp$ in Eq. (1) affects only the Green's functions and can be performed exactly. It replaces the product of the two Green's functions by $G_{k_\parallel}^{-1}(\omega'_m)$. The integration over $k_\parallel$ is generally more involved as for typical $k_\parallel$, the relative deviations of $F_k^{full}(\omega)$ and the scaling function Ψ_k in the fermionic self-energy from their values at a hot spot are both $O(1)$. These deviations are, however, completely irrelevant for the understanding of the physics of the instability, and to proceed with the analytic treatment we assume now that for typical $k_\parallel$, $\Psi_k \approx 1$ and $F_k(\omega) \approx F_{k_{hs}}(\omega)$. Under this assumption, the integration over $k_\parallel$ involves only the spin susceptibility and can also be performed exactly. This last integration replaces $\chi(q, \Omega_m)$ by a local susceptibility $\chi_l(\Omega_m) = \chi_0\xi/(2\sqrt{1 + |\Omega_m|/\omega_{sf}})$.

Observe that the consequences of taking the $N = \infty$ limit are the same as of the Migdal theorem for phonon-mediated superconductors: one can (i) neglect vertex corrections and (ii) explicitly integrate over momentum in the gap equation. From this perspective, our $N = \infty$ analysis of the spin-mediated pairing is analogous to the Eliashberg analysis for conventional superconductors [12].

Combining the results of momentum integration in (1) we obtain, setting $F^{(0)}(\omega_m) = 0$

$$F^{full}(\bar{\omega}_m) = \pi\bar{T}\sum_{\omega_n} \frac{F^{full}(\bar{\omega}_n)}{Z(\bar{\omega}_n)|\bar{\omega}_n|}\ \frac{\lambda}{\sqrt{1 + |\bar{\omega}_m - \bar{\omega}_n|}} \quad (3)$$

where $\bar{T} = T/\omega_{sf}$, and $\bar{\omega}_m = \omega_m/\omega_{sf}$.

We now analyse this equation. Consider first what happens at moderate couplings when $\lambda \leq 1$ (but still, $\xi \gg 1$). In this situation, the fermionic self-energy is irrelevant, and the pairing problem is purely conventional in the sense that (i) at small frequencies the r.h.s. of Eq.(3) is logarithmical, (ii) the upper cutoff for the logarithmical behavior is provided by the frequency dependence of the effective interaction, and (iii) at relevant frequencies, the fermionic factor $Z(\omega)$ is weakly dependent on frequency and can be well approximated by its zero frequency value $Z(0) = 1 + \lambda$. Consequently, the pairing instability temperature T_{ins} has the same form as in the McMillan solution for conventional superconductors [13], i.e., to a good accuracy $T_{ins} \propto \omega_{sf}\ e^{-(1+\lambda)/\lambda}$.

Consider now $\lambda \gg 1$. This limit requires special care as was first indicated in [14]: a simple extention of the weak/moderate coupling approach to $\lambda \gg 1$ yields an erroneous result for T_{ins}. The point is that for large λ, the mere reduction of the pairing interaction above ω_{sf} is not sufficient to eliminate the contribution to the instability from the states with frequencies larger than ω_{sf}. One also has to neutralize the large overall λ factor in the r.h.s. of Eq.(3). At $\omega < \omega_{sf}$, this overall λ is neutralized by Z. However, above ω_{sf}, Z decreases as $Z \sim \lambda(\omega_{sf}/|\omega|)^{1/2}$. In this situation, the contribution to the r.h.s. of Eq.(3) from frequencies larger than ω_{sf} may sweep the pairing instability to a temperature $T_{ins} \gg \omega_{sf}$. To illustrate this we introduce the effective dimensionless coupling constant $\tilde{\lambda} = \lambda(\omega_{sf}/(\pi T))^{1/2}$ and rewrite Eq. (3) at $T \gg \omega_{sf}$ as

$$F^{full}_m = F^{(0)}_m + \gamma \sum_{n \neq m} \frac{F^{full}_n}{\sqrt{2|n-m|}\sqrt{|2n+1|}}\ \frac{\tilde{\lambda}}{2\tilde{\lambda} + \sqrt{|2n+1|}} \quad (4)$$

A fictitious parameter γ (= 1 in our case) is introduced here for the subsequent perturbative analysis of this equation. We see that $\tilde{\lambda}$ is the only parameter in Eq.(4). On general grounds one might expect that the instability occurs at $\tilde{\lambda} = O(1)$, i.e., at $T_{ins} \propto \omega_{sf}\lambda^2$ which is parametrically larger than ω_{sf}. (For phonon-mediated superconducivity, the pairing interaction decreases inversely proportional to the square of the frequency and similar reasoning yields $T_c \propto \sqrt{\lambda}\theta_D$ where θ_D is the Debye frequency [14].)

The above argumentation is however only suggestive, and it is a'priori unclear whether Eq. (4) has a nontrivial solution for any $\tilde{\lambda}$. Furthermore, $T_{ins} \sim \omega_{sf}\lambda^2$ would imply that the pairing is dominated by frequencies where the fermionic excitations display a fully incoherent quantum-critical behavior, $G^{-1} \propto \sqrt{\omega}$, i.e., this mechanism is qualitatively different from the pairing in a Fermi liquid.

To get further insight into the problem, we analysed Eq.(4) for various γ. We found that for small γ, when perturbation theory is valid, Eq. (4) allows only solutions with $F^{full}(\omega) \propto F^{(0)}$. This results from the fact that although the kernel in Eq.(4) has a $1/\omega_n$ form typical for a pairing problem, and gives rise to logarithms in the perturbation theory, it depends on the transferred rather than the total frequency in the Cooper channel. As a result, even at $T = 0$ (when $\tilde{\lambda} = \infty$), the summation of the logarithmical series yields a power-law behavior $F^{full}_m \propto F^{(0)}/|\omega_m|^{\gamma/2}$ rather than a divergence at some finite ω_m.

It turns out, however, that the convergence of the perturbation theory is confined to $\gamma < \gamma_{cr} \approx 0.223$, which is much smaller than our $\gamma = 1$. At $\gamma > \gamma_{cr}$, there exists a critical value of $\tilde{\lambda}$ when Eq. (4) has a nontrivial solution for $F^{full}(\omega)$ even if $F^{(0)} = 0$. Near $\gamma = \gamma_{cr}$, $\tilde{\lambda}_{cr} \propto e^{\pi/\beta}$ where $\beta = 1.2(\gamma - \gamma_{cr})^{1/2}$. We see that for $\gamma = 1$, the attraction between fully incoherent fermions is capable to produce a pairing instability at $T_{ins} \propto \lambda^2\omega_{sf} \sim \bar{g}/N$, as we conjected above, but this result has a non perturbative origin.

To check this analysis, we solved our original Eq. (3) numerically for various λ. The results are presented in Fig. (2). To identify the contribution from incoherent fermionic states, we present the results for T_{ins} and for T^{coh}_{ins}, which is the solution of Eq.(3) with $Z = \lambda + 1$ as in the low frequency region. This Z accounts for strong renormalization of the quasiparticle mass, but leaves fermionic excitations fully coherent. With a reasonable numerical accuracy, we found $T^{coh}_{ins} \approx 1.6\omega_{sf}e^{-(1+\lambda)/\lambda}$. The ratio of the two temperatures measures the relative effect of the fermionic incoherence on T_{ins}. When this ratio is close to 1, the instability temperature is predominantly determined by coherent excitations, while when it is large, T_{ins} is mostly determined by the incoherent part of the fermionic spectral function. We see from Fig. (2) that up to $\lambda \sim 1$, $T_{ins}/T^{coh}_{ins} \approx 1$, but at larger λ this ratio goes up and eventually behaves as λ^2. In the latter case we found $T_{ins} \approx 0.3\lambda^2\omega_{sf}$. Notice that while $\omega_{sf} \propto \xi^{-2}$ vanishes at $\xi = \infty$, the product $\lambda^2\omega_{sf} = 9\bar{g}/(8\pi N)$ is independent on ξ. This is consistent with the fact that the pairing is produced by fermions which display a quantum-critical behavior.

The analysis of the system behavior below T_{ins} requires one to solve a set of three coupled integral equations for the fermionic self-energy, spin-polarization operator, and the anomalous vertex. Setting this aside for a separate publication, we merely argue here that although $F^{full}(\omega) \neq 0$ below T_{ins}, it still vanishes at zero frequency and therefore does not give rise to a nonzero value of the superconducting order parameter $\int dt' < c(t)\chi_l(t-t')c(t') > \ \propto F^{full}(\omega = 0)$. Indeed, a nonzero $F^{full}(0)$ would give rise to the $1/|\omega|$ behavior of the local susceptibility [10] which for $\xi = \infty$ causes divergences in the Eliashberg equations and thus prevents any solution with $F(0) \neq 0$.

We conject that at finite ξ, a nonzero value of $F^{full}(0)$,

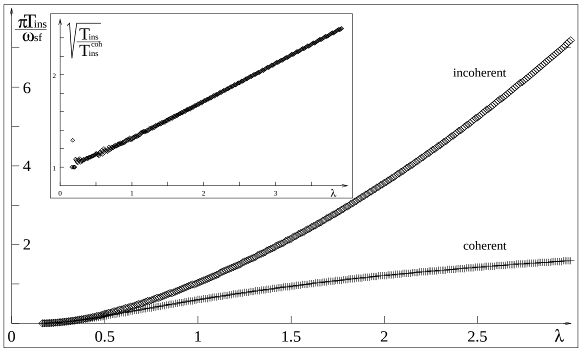

FIG. 2. The results of the numerical solution of Eq (3) for different values of the coupling constant λ. The actual T_{ins} and a fictitious T_{ins}^{coh} are obtained by eliminating the incoherent part of the fermionic self-energy. The inset shows the ratio of the two temperatures and a fit to λ^2 behavior at large λ.

and hence a true superconductivity, appears only when coherent fermionic excitations become dominant. This yields $T_c \sim T_{ins}^{coh}$, and the pseudogap phase between T_{ins} and T_{ins}^{coh}. We see from Fig. 2 that T_{ins}^{coh} saturates at around $0.4\omega_{sf}$. Near optimal doping $\omega_{sf} \sim 20-30meV$ [1]. This yields $T_c \sim 100K$ consistent with the data. Notice that $T_c/\xi^{-1} \approx (0.08/\lambda)v_F \ll v_F$. Recent comparison of T_c with the width of the neutron scattering peak in momentum space [15] also yielded $T_c \ll v_F$. Finally, observe that T_{ins} deviates from T_{ins}^{coh} before T_{ins}^{coh} saturates, i.e. in our scenario, the pseudogap behavior emerges well above optimal doping.

We next discuss the momentum dependence of $F^{full}(k,\omega)$ at $\omega \neq 0$ at T_{ins}. This momentum dependence is likely to mimic that of a pseudogap at $T \lesssim T_{ins}$ [16]. Simple estimates show that away from a hot spot, $F^{full}(k)$ remains flat up to $|k-k_{hs}| \sim k_{typ}$ where $k_{typ} \sim \xi^{-1}$ for $\lambda \leq 1$, and $k_{typ} \sim \bar{g}/(v_F k_F)$ for $\lambda \gg 1$. At further deviations from a hot spot, the gap decreases and near the zone diagonal vanishes as $F(k) \propto |k_x - k_y|$ due to a competition between two hot spots with different sign of the gap. This behavior is consistent with the photoemission data [17].

Finally we discuss how our work is connected to earlier studies. The Eliashberg-type equations we solved are almost the same as used in previous extensive numerical and analytical studies [1,3–5], and particularly in [2] - our $1/N$ expansion is just a way to justify the neglect of vertex corrections. Our key finding is the discovery that the superconducting problem at large ξ and *in the presence of hot spots* is universal and does not depend on the form of the pairing potential at lattice scales. This physics was not detected in most of earlier works [1–4] which considered moderate $\xi \sim 2-3$, and the Fermi surface without hot spots. Eliashberg equations for large ξ and for the Fermi surface with hot spots have recently been solved numerically by Monthoux and Lonzarich [5]. They found that for large couplings, T_{ins} scales as $\omega_{sf}\xi^2$ and saturates at a finite value at $\xi = \infty$. This fully argees with our result for T_{ins}. We argue, however, that at $\xi \to \infty$, T_{ins} is the onset temperature for the pseudogap behavior ($T_{ins} = T^*$), rather than T_c.

To summarize, in this paper we have demonstrated that for a Fermi surface with hot spots, the pairing problem below optimal doping is self-consistently confined to low energies and fermionic momenta near k_{hs}, and becomes a universal one. At weak/moderate coupling, the pairing instability is determined by coherent fermionic excitations, and the instability temperature T_{ins} coincides with T_c and scales with the spin-fluctuation frequency ω_{sf}. Near optimal doping, we obtained $T_c \sim 0.4\omega_{sf} \sim 100K$ consistent with the data. At strong coupling we found that the pairing problem is dominated by incoherent fermionic excitations. We have shown that in this situation, $T_{ins} \propto \lambda^2\omega_{sf}$ tends to a nonzero value at $\xi = \infty$. We argued, however, that this T_{ins} is the onset temperature for the pseudogap behavior, while the actual $T_c \propto \omega_{sf}$ and vanishes when $\xi = \infty$. The pseudogap below T_{ins} has a $d_{x^2-y^2}$ symmetry, but it is mostly concentrated near hot spots.

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[1] Ph. Monthoux and D. Pines, Phys. Rev. B **47**, 6069 (1993);
[2] Ph. Monthoux and D.J. Scalapino, Physica B, **199& 200**, 294 (1994), see also Phys. Rev. Lett **69**, 961 (1992).
[3] A.J.Millis, Phys. Rev. B **45**, 13047 (1992).
[4] St. Lenck, J. P. Carbotte and B. Dynes, Phys. Rev. B **49**, 6933 (1994); see also St. Lenck and J.P. Carbotte, Phys. Rev. B **49**, 4176 (1994).
[5] Ph. Monthoux and G.G. Lonzarich, Phys. Rev. B **59**, 14598 (1999).
[6] D.J. Scalapino, Phys. Rep. **250**, 329 (1995); D. Pines, Z. Phys. B **103**, 129 (1997) and references therein.
[7] see, e.g., J. Annett, N. Goldenfeld and S. Renn in *Physical properties of high temperature superconductors*, ed. by D. Ginsberg (World Scientific, NJ 1990).
[8] E.M. Lifshitz and L.P. Pitaevskii, Statistical Physics, p.2
[9] A. Chubukov, Europhys. Lett. **44**, 655 (1997); A. Chubukov and D. Morr, Phys. Rev. Lett. **81**, 4716 (1998);
[10] Ar. Abanov and A. Chubukov, Phys. Rev. Lett., **83**, 1652 (1999).
[11] A. Chubukov and D. Morr, Phys. Rep., **288**, 355 (1997).
[12] G.M. Eliashberg, Sov. Phys. JETP **11**, 696 (1960).

[13] W.L. McMillan, Phys. Rev. **167**, 331 (1968).
[14] P.B. Allen and R.C. Dynes, Phys. Rev. B **12**, 905 (1975).
[15] A. V. Balatsky and P. Bourges, Phys. Rev. Lett, **82**, 5337 (1999).
[16] V.L. Pokrovsky, Sov. Phys. JETP **12**, 447 (1961).
[17] J. Mesot et al, cond-mat/9812377; R. Gatt et al, cond-mat/9906070.