

## Instability of the Quantum-Critical Point of Itinerant Ferromagnets

Andrey V. Chubukov,<sup>1</sup> Catherine Pépin,<sup>2</sup> and Jerome Rech<sup>2,3</sup>

<sup>1</sup>*Department of Physics, University of Wisconsin–Madison, 1150 University Avenue, Madison, Wisconsin 53706-1390, USA*

<sup>2</sup>*SPhT, CEA-Saclay, L'Orme des Merisiers, 91191 Gif-sur-Yvette, France*

<sup>3</sup>*Center for Materials Theory, Rutgers University, Piscataway, New Jersey 08855, USA*

(Received 18 November 2003; published 9 April 2004)

We study the stability of the quantum-critical point for itinerant ferromagnets commonly described by the Hertz-Millis-Moriya (HMM) theory. We argue that in  $D \leq 3$  long-range spatial correlations associated with the Landau damping of the order parameter field generate a universal *negative*, nonanalytic  $|q|^{(D+1)/2}$  contribution to the static magnetic susceptibility  $\chi_s(q, 0)$ , which makes HMM theory unstable. We argue that the actual transition is either towards incommensurate ordering, or first order. We also show that singular corrections are specific to the spin problem, while charge susceptibility remains analytic at criticality.

DOI: 10.1103/PhysRevLett.92.147003

PACS numbers: 74.25.-q, 75.50.-y

The critical behavior of itinerant paramagnets at the onset of ferromagnetic ordering is the subject of intensive experimental [1] and theoretical [2,3] studies. Of particular interest is the existence of a ferromagnetic second-order transition at low temperatures, and the emergence of superconductivity near the phase boundary. The critical theory of itinerant ferromagnets was developed by Hertz, Millis, and Moriya [4] (HMM) and is based on the action of the form

$$S = \int d^d q d\omega_m \left[ \xi^{-2} + q^2 + \frac{|\omega_m|}{|q|} \right] \phi_{q,\omega}^2 + b_4 \phi^4 + b_6 \phi^6 + \dots, \quad (1)$$

where  $\phi$  is a bosonic field associated with the order parameter, and  $\xi$  is the correlation length, which diverges at the phase transition. The two assumptions behind Eq. (1) are that the  $b_{2n, n \geq 2}$  terms are nonsingular and can be approximated by constants, and that the static spin susceptibility has a regular  $q^2$  momentum dependence. The HMM theory has been successfully applied to explain quantum-critical behavior in a number of materials [1]; however, its key assumptions have been questioned in recent studies [5,6]. The analyticity of the  $b_{2n}$  terms was analyzed in detail for an antiferromagnetic transition where it was demonstrated that, for  $D \leq 3$ , all  $b_{2n}$  prefactors do have nonanalytic pieces which depend on the ratio between momenta and frequencies of the  $\phi$  fields [5]. For a 2D antiferromagnetic quantum-critical theory (2D fermions and 2D spin fluctuations) these nonanalytic terms give rise to singular vertex corrections [5]. For a 2D ferromagnetic case, nonanalytic terms in  $b_{2n}$  are still present; however, we found that they do not give rise to an anomalous exponent in the spin susceptibility for  $D > 1$  and therefore are not dangerous.

In this Letter, we question another key assumption of the HMM theory, namely, that the momentum dependence of the static spin propagator is analytic at small  $q$ . This assumption is based on the belief that in itinerant

ferromagnets the  $q$  dependence of the  $\phi^2$  term comes solely from fermions with high energies, of order of  $E_F$ , in which case the expansion in powers of  $(q/p_F)^2$  should generally hold for  $q \ll p_F$ . This reasoning was disputed in Refs. [6,7]. These authors considered a static spin susceptibility  $\chi_s(q)$  in a weakly interacting Fermi liquid, i.e., well away from a quantum ferromagnetic transition, and argued that for  $D \leq 3$  and arbitrary small interaction, the small  $q$  expansion of  $\chi_s(q)$  begins with a nonanalytic  $|q|^{D-1}$  term. This nonanalyticity originates from a  $2p_F$  singularity in the particle-hole polarization bubble [6–8] and comes from *low-energy* fermions with energies of the order of  $v_F q \ll E_F$ . In Ref. [6], it was further argued that within the RPA, the nonanalyticity in  $\chi_s(q)$  gives rise to the emergence of a nonanalytic  $|q|^{D-1} \phi_{q,\omega}^2$  term in Eq. (1). Furthermore, the prefactor of this term turns out to be negative, which signals the breakdown of the continuous transition to ferromagnetism.

The weak point of this reasoning is that within the RPA one assumes that fermionic excitations remain coherent at the quantum-critical point (QCP). Meanwhile, it is known [9] that, upon approaching the QCP, the fermionic effective mass  $m^*$  diverges as  $\log \xi$  in  $D = 3$  and as  $\xi^{3-D}$  in smaller dimensions. We checked that  $m/m^*$  appears as a prefactor of the  $|q|^{D-1}$  term; this term *vanishes* at the QCP. Does this imply that Eq. (1) is valid at the transition? Not necessarily, since one has to verify explicitly whether or not the divergence of  $m^*$  completely eliminates a nonanalyticity in  $\chi_s(q)$  or just makes the nonanalytic term weaker than away from QCP. If the latter is true, the nonanalytic term can still be much larger than  $q^2$  at small  $q$  at criticality.

In this Letter, we report explicit calculations which show that the nonanalytic term is still present at the QCP and accounts for the breakdown of the HMM theory. We first consider the problem in the Eliashberg approximation and argue that this approximation leads to a non-Fermi liquid behavior at the QCP, but the magnetic susceptibility  $\chi_s(q)$  remains analytic in  $q$ . We then

demonstrate that the corrections to the Eliashberg theory are singular and give rise to a universal contribution to  $\chi_s^{-1}(q)$  in the form  $q^\beta$ , with  $\beta \leq 2$  for  $D \leq 3$  and a *negative* prefactor. We show that this universal contribution makes the ferromagnetic QCP unstable and gives rise to either a continuous transition into a spiral state or a first-order transition into a ferromagnet.

*The model.*—The starting point for our theory is a low-energy effective spin-fermion Hamiltonian obtained by integrating the fermions with energies between fermionic bandwidth  $W$  and  $\Lambda \ll W$  out of the partition function [3,9]:

$$\mathbf{H} = \sum_{p,\alpha} v_F(p - p_F) c_{p,\alpha}^\dagger c_{p,\alpha} + \sum_q \chi_{s,0}^{-1}(q) \mathbf{S}_q \mathbf{S}_{-q} + g \sum_{p,q} c_{p+q,\alpha}^\dagger \boldsymbol{\sigma}_{\alpha,\beta} c_{p,\beta} \mathbf{S}_q. \quad (2)$$

Here  $\mathbf{S}_q$  describe collective bosonic degrees of freedom in the spin channel, and  $g$  is the residual spin-fermion coupling. The upper cutoff  $\Lambda$  is roughly the scale up to which fermionic dispersion can be linearized near the Fermi surface. The bare spin susceptibility  $\chi_{s,0}(q)$  is assumed to be analytic at  $q \ll p_F$  and have an Ornstein-Zernike form  $\chi_{s,0} = \chi_0/(\xi^{-2} + q^2)$ . The coupling and  $\chi_0$  appear in the theory only in the combination  $\bar{g} = g^2 \chi_0$ .

The perturbation theory for Eq. (2) is an expansion into two parameters [3]:

$$\alpha = \frac{3^{3/4} \bar{g}}{4\pi E_F} \quad \text{and} \quad \lambda = \frac{3}{4\pi v_F \xi^{-1}} \bar{g} \sim \alpha(\xi p_F), \quad (3)$$

where  $E_F$  is the electron's Fermi energy and the prefactor in  $\alpha$  is chosen for future convenience. We assume that  $\bar{g}$  is small compared to  $E_F$ ; i.e.,  $\alpha$  is a small parameter [3,10]. That  $g \ll E_F$  is in line with the very idea of a separation between high-energy and low-energy physics. At the same time, near a ferromagnetic QCP,  $\xi$  diverges, and the dimensionless coupling  $\lambda \propto \xi$  is large. Fortunately, one can solve explicitly the strong coupling problem in  $\lambda$  while keeping the zeroth order in  $\alpha$  [3,9]. This amounts to neglecting the vertex correction  $\delta g/g$  and the momentum-dependent piece in the self-energy  $\Sigma(p)$ , which are of next order in  $\alpha$ . The analogous theory for electron-phonon interaction is known as Eliashberg theory [11].

*Eliashberg theory.*—The Eliashberg theory for a ferromagnetic quantum-critical point has been described in detail in the literature [3,9]. The results are presented here for  $D = 2$ , but their validity holds for general  $D \leq 3$ . The elements of the theory are coupled fermionic and bosonic self-energies  $\Sigma(\omega_m)$  and  $\Pi(q, \omega_m) = \Phi(\omega_m)/|q|$ , respectively [3]  $\{G^{-1}(p, \omega_m) = i[\omega_m + \Sigma(\omega_m)] - v_F(p - p_F)$ ;  $\chi_s(q, \omega_m) = \chi_0/[q^2 + \xi^{-2} + \Pi(q, \omega_m)]\}$ . The self-consistent solution yields  $\Phi(\omega_m) = \gamma|\omega_m|$ , where  $\gamma = \bar{g} p_F/(\pi v_F^2)$ , and

$$\Sigma(\omega_m) = \lambda \omega_m f\left(\frac{(p_F \xi)^3 \omega_m \bar{g}}{E_F^2}\right), \quad (4)$$

where  $f(0) = 1$ ,  $f(x \gg 1) = (32\pi/3\sqrt{3}x)^{1/3}$ , and  $E_F = p_F v_F/2$ . At the QCP,  $\xi = \infty$  and  $\Sigma(\omega_m) = \omega_m^{2/3} \omega_0^{1/3}$ , where  $\omega_0 = \alpha^2 E_F$  is the typical bosonic frequency of our problem. Note that  $\omega_0 \ll E_F$ . The  $\omega^{2/3}$  dependence of  $\Sigma$  implies that at the QCP, the Fermi-liquid description is broken down to the lowest energies. At the same time, Eliashberg theory reproduces the form of the spin propagator from Eq. (1),  $\chi_s(q, \omega_m) \propto (\xi^{-2} + q^2 + \gamma|\omega_m|/q)^{-1}$ . Alternatively speaking, in the Eliashberg approximation, the Fermi liquid is destroyed at QCP, but the magnetic transition remains continuous, and  $\chi_s(q)$  is analytic.

*Beyond Eliashberg theory.*—The validity of the Eliashberg theory is generally based on Migdal's theorem that states that vertex correction  $\delta g/g$  and  $\Sigma(p)$  are small when the bosonic velocity  $v_B$  is much smaller than  $v_F$ . In our case, *a posteriori* analysis of typical bosonic and fermionic momenta which contribute to  $\Phi(\omega)$  and  $\Sigma(\omega)$  within the Eliashberg theory shows that for the same  $\omega \sim \omega_0$ , typical  $q_B \sim (\gamma \omega_0)^{1/3} \sim \alpha p_F$ , while typical fermionic momenta  $|p - p_F| \sim \omega_0/v_F \sim \alpha^2 p_F$  are much smaller. This implies that for  $\alpha \ll 1$ , the effective bosonic velocity  $v_B \sim \alpha v_F$  is much smaller than  $v_F$ ; i.e., Migdal's theorem is valid. To verify this, we computed the corrections to  $\partial \Sigma(p)/\partial p$  and to the static vertex in the limit of zero bosonic momentum and for external fermions at the Fermi surface and at zero frequency and indeed found  $\alpha^{1/2}$  smallness.

Still, this does not immediately imply that  $\chi_s(q)$  remains analytic beyond Eliashberg theory. To verify this, we have to go beyond the limit of zero momentum, evaluate the correction to the static susceptibility at a finite  $q$ , and check whether it remains analytic in  $q$ . Let us do this. Consider, e.g., the diagram in Fig. 1(a) with the insertion of  $\Sigma(p)$  into the particle-hole bubble. Expanding this diagram to order  $q^2$  we find that it scales as  $q^2 I$ , where

$$I \propto \alpha^2 \frac{E_F^3}{\gamma^{1/3}} \int \frac{dl_x d\Omega |\Omega|^{2/3}}{(v_F l_x - i|\Omega + \Sigma(\Omega)|)^5} S\left(\frac{l_x^2}{(|\Omega| \gamma)^{2/3}}\right). \quad (5)$$

Here  $l^2 = l_x^2 + l_y^2$ , and  $S(\dots) \sim (|\Omega| \gamma)^{1/3} \int dl_y / (l^2 + |\Omega| \gamma / |l|)$  is, up to a prefactor, the boson propagator integrated along the Fermi surface. In the two limits,  $S(0) = O(1)$  and  $S(x \gg 1) \propto x^{-1/2}$ . The integrand for  $I$  has a highly degenerate pole in the upper half-plane of  $l_x$ , at  $v_F l_x \propto |\Omega + \Sigma(\Omega)|$ . This degenerate pole can be avoided by closing the integration contour in the lower

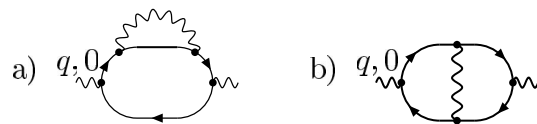


FIG. 1. The two lowest-order diagrams for the corrections to the static spin susceptibility. The self-energy insertion is  $\Sigma(p)$ ; the frequency dependent piece  $\Sigma(\omega_m)$  is already incorporated into “zero-order” Eliashberg theory.

half-plane of  $l_x$ , in which case the nonvanishing part in  $I$  comes from the nonanalyticity in  $S(x)$ . This physically implies that fermions undergo forced vibrations at typical bosonic frequencies. The issue then is what is the typical bosonic  $x$  for the nonanalyticity in  $S(x)$ . By analogy with the electron-phonon problem, one could expect that corrections to the Eliashberg theory come from the processes in which typical bosonic frequencies are near bosonic mass shell. Then typical  $l_x$  are of order  $(\Omega\gamma)^{1/3}$  as in the Eliashberg theory; i.e., typical  $x$  in  $S(x)$  are of order 1. Substituting this typical  $x$  into (5), we obtain  $I \sim \alpha^{1/2}$ , i.e., a small and an analytic correction to the static susceptibility  $\delta\chi_s(q) \propto q^2\alpha^{1/2}$ . However, a careful examination of the scaling function  $S(x)$  reveals that it is *nonanalytic* already at the smallest  $x$ , such that there is another contribution to  $I$  from bosons vibrating at typical  $l_x$  comparable to those near the *fermionic* mass shell. Indeed, expanding  $S(x)$  in powers of  $x$  at  $x \ll 1$  we obtain, to logarithmical accuracy,

$$S(x) = 1 - \frac{3\sqrt{3}}{8\pi} \frac{l_x^2}{(\gamma|\Omega|)^{2/3}} \log[l_x^2]. \quad (6)$$

The  $\log l_x^2$  term is the most important here—its presence implies that  $S(x)$  possesses a branch cut along the imaginary axis of  $l_x$  down to the smallest  $l_x$ . One can easily make sure that this logarithmic singularity is the consequence of the  $\Omega/|l|$  nonanalyticity of the Landau damping piece in the bosonic propagator, i.e., of the existence of the long-range spatial component of the dynamical susceptibility. Substituting the small  $x$  form of  $S(x)$  into (5) and evaluating integrals, we find a much larger, divergent contribution to  $I$  which behaves as  $|q|^{-1/2}$ . This implies that the low- $q$  expansion of  $\delta\chi_s$  is actually nonanalytic in  $q$  and begins as  $q^{3/2}$ . Performing an explicit calculation without expanding in  $q$ , and combining the results from the two diagrams for  $\chi(q)$  in Fig. 1 we obtain for the static spin susceptibility

$$\chi_s(q) = \frac{\chi_0}{q^2 - 0.17|q|^{3/2}p_F^{1/2}}. \quad (7)$$

Observe that the  $q^{3/2}$  term in (7) is not small in  $\alpha$ . This is not surprising, since the only excitations involved are those near the fermion mass shell, while  $\alpha$  measures how soft the mass shell bosons are compared to the mass shell fermions. Note in passing that the self-energy insertion in the diagram of Fig. 1(a) is not a double counting, since the correction to static susceptibility comes only from the momentum-dependent piece in the self-energy. The frequency-dependent  $\Sigma(\omega)$  is already incorporated at the zero-order level, and it does not affect  $\chi_s(q)$ .

The nonanalyticity of the correction to the susceptibility and the disappearance of  $\alpha$  could be also detected by analyzing vertex corrections away from the limit of  $q = 0$  and zero fermion frequency  $\omega$ . Indeed, evaluating  $\delta g(q, \omega)/g$  at finite  $q$  and  $\omega$ , we obtain

$$\delta g(q, \omega) \propto g\alpha^{1/2}\psi\left(\frac{m\bar{g}}{q^2}, \frac{\omega}{E_F}\right), \quad (8)$$

where  $\psi(x \ll 1, y \ll 1) \propto \sqrt{x} \ll 1$ , and  $\psi(x \gg 1, y \ll 1) = 1 + A(\log y)/\sqrt{x}$ ,  $A = O(1)$ . We see that, although  $\psi(x, y) \leq 1$  (i.e., vertex correction is small in  $\alpha^{1/2}$ ), the small  $q$  expansion of  $\delta g(q)/g$  begins as  $\alpha^{1/2}|q||\log \omega|/\sqrt{m\bar{g}} \sim |q||\log \omega|/p_F$ ; i.e., it is nonanalytic in  $q$  and in  $\omega$ , and the  $|q \log \omega|$  term does not contain  $\alpha^{1/2}$ . This singular term again comes from fermions near their own resonance and can be traced back to the nonanalyticity of the Landau damping term. Substituting this  $\delta g(q, \omega)$  into the susceptibility diagram and performing computations we find that  $\log \omega$  dependence makes  $\delta\chi_s(q)$  nonzero in the static limit (without it, the correction would be proportional to  $\Phi(\omega)$  and vanish at  $\omega = 0$ ), and the overall power of  $q$  is  $|q|$  from the vertex correction times the ratio of typical  $\omega$  and  $v_F l_x$ . As typical  $v_F l_x \sim \Sigma(\omega) \propto \omega^{2/3}$ , this ratio yields  $\omega^{1/3} \propto (l_x)^{1/2} \sim q^{1/2}$ ; hence the overall power in  $\delta\chi_s(q)$  is  $q^{3/2}$ , as above.

What happens with higher-order terms? We analyzed higher-order particle-hole insertions into the susceptibility bubble and found that they add only small,  $O(\alpha^{1/2})$ , corrections to the  $q^{3/2}$  term in (7) since at least one internal momentum is near the boson mass shell. Higher-order particle-particle insertions give rise to  $O(1)$  corrections to the  $q^{3/2}$  term and, in principle, should be included. However, the nonsmallness of particle-particle insertions is not specific to our problem and is, in fact, customary in Eliashberg-type theories [2,5,3,11,12,10]. Particle-particle vertex corrections are known to give rise to a pairing instability close to the ferromagnetic QCP at  $T_c = O(\omega_0)$ , which implies that there exists a dome around the QCP where the normal state analysis is invalid. However, a ferromagnetic  $T_c$  has an order of magnitude of roughly  $T_c \leq 0.015\omega_0$  [3]. Hence the typical bosonic momenta for the pairing are of order  $0.01\alpha p_F$ , much smaller than typical  $q$  in Eq. (7). Because of this separation of energy scales, we do not expect particle-particle insertions to be relevant.

Equation (7) is the central result of our Letter. It shows that the static susceptibility becomes negative around a ferromagnetic QCP at  $q < q_1 = 0.029p_F$ . Although the prefactor is small,  $q_1$  is parametrically larger than the typical fermion and boson momenta that contribute to the fermion self-energy in the quantum-critical regime ( $|k - k_F| \sim \alpha^2 p_F$  and  $q_B \sim \alpha p_F$ , respectively). This implies that the whole quantum-critical region is, in fact, *unstable*. Alternatively speaking,  $Z = 3$  quantum-critical behavior at a ferromagnetic transition is internally destroyed by infrared singularities associated with the nonanalytic momentum dependence of the Landau damping.

A negative static susceptibility up to a finite  $q$  implies either that the instability occurs when  $\xi^{-1}$  is still finite, into an incommensurate state with a finite  $|q| \sim q_1$  [13], or the ordering is commensurate, but the magnetic transition is of the first order. Which of the two scenarios

holds remains to be studied. We also verified that away from the QCP, when  $\xi$  is finite, the  $q^{3/2}$  term transforms into  $|q|\xi^{-1}$  at the smallest  $q$ . The interpolation formula between the  $q^{3/2}$  and  $|q|$  forms is rather involved and we do not present it here.

For arbitrary  $D < 3$ , our calculations yield the “correction” to  $\chi_s^{-1}$  in the form  $|q|^{(D+1)/2}$ , again with a negative prefactor. In 3D it scales as  $q^2 \log|q|$ . Hence, HMM theory is destroyed for all  $D \leq 3$ . For  $D > 3$ , the correction term is smaller than  $q^2$ , and HMM theory survives.

*Charge susceptibility and gauge theory.*—We repeated the same calculations for the charge susceptibility and found in  $D = 2$  that the singular  $|q|^{3/2}$  terms from the two diagrams in Fig. 1 cancel each other, and the  $q^2$  behavior survives. The same holds for the propagator of the gauge field—singular  $|q|^{3/2}$  terms from self-energy and vertex correction insertions into the particle-hole bubble again cancel each other. We verified the cancellation of non-analytic terms for the second-order diagrams as well. Furthermore, we verified in two orders of perturbation that the momentum dependence of  $\chi_c(q, 0)$  comes in powers of  $(q/p_F)^2$ ; i.e., it is entirely due to the existence of a curvature of the dispersion. Physically, the distinction between spin and charge susceptibilities is in that the  $\Omega/|q|$  singularity in the dynamical particle-hole response function (which generates the  $q^{3/2}$  nonanalyticity) is sensitive to a magnetic field but is insensitive to the change of the chemical potential. Hence, the singularity shows up in the total spin response but does not appear in the total charge response [14]. Mathematically, this distinction is due to the presence, in the spin case, of the Pauli matrices in the vertices of the diagrams in Fig. 1, such that the self-energy and vertex correction diagrams do not cancel each other. Note that the cancellation or noncancellation of the singularities is not directly related to the conservation laws for charge and spin susceptibilities, since conservation laws require only that  $\chi_c(q = 0, \omega)$  and  $\chi_s(q = 0, \omega)$  vanish but impose no constraints on the forms of the susceptibilities in the other limit of finite  $q$  and zero frequency. More specifically, Fermi-liquid relations between  $\chi_{s,c}(q \rightarrow 0, \omega = 0)$  and  $\chi_{s,c}(q = 0, \omega \rightarrow 0)$  imply [15] that  $\chi_{s,c}(q \rightarrow 0, \omega = 0) \rightarrow Cst$  but impose no formal constraints on the form of the actual  $q$  dependence in  $\chi_{s,c}(q, \omega = 0)$  at finite  $q$ .

To summarize, we studied the stability of the quantum-critical point for itinerant ferromagnets, in the limit when the spin-fermion coupling is much smaller than the bandwidth, and one would naively expect Eliashberg theory to be valid. Within Eliashberg theory, the Fermi-liquid is destroyed at criticality, but the magnetic transition is still continuous and is described by bosonic Hertz-Millis-Moriya action. We demonstrated, however, that in  $D \leq 3$ , long-range spatial correlations associated with the Landau damping of the bosonic order parameter field break the Eliashberg theory and give rise to a universal, *negative*, nonanalytic  $|q|^{(D+1)/2}$  contribu-

tion to the static magnetic susceptibility  $\chi_s(q, 0)$ . This term makes the continuous critical theory unstable. We argued that the actual transition is either towards incommensurate ordering or first-order into a commensurate state. We also demonstrated that singular corrections are specific to the spin problem, while charge susceptibility remains analytic at criticality.

It is our pleasure to thank A. Abanov, I. Aleiner, A. Andreev, B. Altshuler, D. Belitz, P. Coleman, S. Das Sarma, A. Georges, L. Glazman, A. Finkelstein, Y. B. Kim, T. Kirkpatrick, D. Maslov, A. Millis, M. Norman, O. Parcollet, I. Paul, R. Ramazashvili, Q. Si, and Th. Vojta for stimulating discussions. The research has been supported by NSF DMR 9979749 (A. V. C.).

- 
- [1] See, e.g., G. R. Stewart, *Rev. Mod. Phys.* **73**, 797 (2001), and references therein.
  - [2] R. Roussev and A. J. Millis, *Phys. Rev. B* **63**, 140504 (2001); Z. Wang, W. Mao, and K. Bedell, *Phys. Rev. Lett.* **87**, 257001 (2001); for related developments, see also P. Coleman, C. Pepin, Q. Si, and R. Ramazashvili, *J. Phys. Condens. Matter* **13**, 723 (2001).
  - [3] A. Chubukov, A. Finkelstein, R. Haslinger, and D. Morr, *Phys. Rev. Lett.* **90**, 077002 (2003).
  - [4] J. A. Hertz, *Phys. Rev. B* **14**, 1165 (1976); A. Millis, *Phys. Rev. B* **48**, 7183 (1993); T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism* (Springer-Verlag, Berlin, New York, 1985).
  - [5] A. Abanov, A. V. Chubukov, and J. Schmalian, *Adv. Phys.* **52**, 119 (2003); Ar. Abanov, A. V. Chubukov, and A. M. Finkelstein, *Europhys. Lett.* **54**, 488 (1999).
  - [6] D. Belitz, T. R. Kirkpatrick, and T. Vojta, *Phys. Rev. B* **55**, 9452 (1997).
  - [7] A. V. Chubukov and D. L. Maslov, *Phys. Rev. B* **68**, 155113 (2003); see also A. Houghton, H.-J. Kwon, and B. Marston, *Adv. Phys.* **49**, 141 (2000); S. Das Sarma and E. H. Hwang, *Phys. Rev. Lett.* **83**, 164 (1999).
  - [8] G. Y. Chitov and A. J. Millis, *Phys. Rev. Lett.* **86**, 5337 (2001); *Phys. Rev. B* **64**, 054414 (2001).
  - [9] B. Altshuler, L. B. Ioffe, and A. J. Millis, *Phys. Rev. B* **52**, 5563 (1995).
  - [10] M. Dzero and L. P. Gorkov, *Phys. Rev. B* **69**, 092501 (2004).
  - [11] G. M. Eliashberg, *Sov. Phys. JETP* **16**, 780 (1963); D. J. Scalapino, in *Superconductivity*, edited by R. D. Parks (Dekker Inc., New York, 1969), Vol. 1, p. 449; F. Marsiglio and J. P. Carbotte, in *The Physics of Conventional and Unconventional Superconductors*, edited by K. H. Bennemann and J. B. Ketterson (Springer-Verlag, Berlin, 2002).
  - [12] A. V. Chubukov, D. Pines, and J. Schmalian, in *The Physics of Superconductors*, edited by K. H. Bennemann and J. B. Ketterson (Springer, New York, 2003), Vol. 1, p. 495.
  - [13] S. A. Brazovskii, *Sov. Phys. JETP* **41**, 85 (1975).
  - [14] D. Belitz and T. R. Kirkpatrick, *Phys. Rev. B* **67**, 024419 (2003); (unpublished).
  - [15] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon Press, Oxford, 1980).