Possibility of a superfluid transition in a slightly nonideal Fermi gas with repulsion

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A transition to a superfluid state with an angular momentum $l = 1$ occurs in a low-density Fermi gas with repulsion.

I. According to the existing theory of superconductivity, in a low-density Fermi gas with a large distance between particles, i.e., in a gas with $|a|p_F \ll 1$, where $a < 0$ is the s-scattering length, the formation and condensation of bound singlet Cooper pairs begin at a temperature $T_{c0} = A\epsilon_F \exp\{-\pi/2p_F|a|\}$ $(A = (\gamma/\pi)(2/e)^{7/3}$; where $\ln\gamma$ is the Euler constant]. As a result, the gas goes into a superfluid state.[1,2] It has customarily been assumed that a low-density Fermi gas with a repulsion between particles $(a > 0)$ remains in a normal phase down to $T = 0$. Our purpose in the present letter is to show that in the case of repulsion a Fermi gas will also go into a superfluid state at a certain temperature $T_{c1} \sim \epsilon_F \exp\{-1(ap_F)^2\}$, but the orbital angular momentum of the relative motion will be $l = 1$ (so the total spin is $S = 1$). We know that a sufficient condition for the onset of superfluidity is that the effective interaction $\tilde{\Gamma}_l$ be of the nature of an attraction for at least a single value of the angular momentum l (Ref. 2). We will show that the condition $\tilde{\Gamma}_1 < 0$ clearly holds in a low-density Fermi gas, even if we have $\tilde{\Gamma}_0 \approx (4\pi/m)a > 0$. Our proof is based on the circumstance that, as Kohn and Luttinger have shown,[3] the presence of a filled Fermi sphere leads to a substantial renormalization of the angular dependence of the effective interaction, so the expansion of the partial components $\tilde{\Gamma}_l$ in the gas parameter—valid for slow particles in vacuum—is no longer valid.

II. A mathematical expression of the instability of a normal state which leads to a superfluid transition is the appearance of a pole in the total vertex Γ which is associated with the situation in which any of the partial components Γ_l becomes infinite. The total vertex Γ is usually determined by the Bethe-Salpeter equation shown graphically in Fig. 1. We can now determine the effective interaction $\tilde{\Gamma}$ explicitly. This interaction is given by the complete set of diagrams which are not cut along the two fermion lines which run in the same direction. Figure 2 shows diagrams which contribute to $\tilde{\Gamma}$ in the first two orders of a perturbation theory in the parameter $\alpha P_F \ll 1$.

In first-order perturbation theory, the effective seed vertex $\tilde{\Gamma}$ is the same as the potential of the two-particle interaction, $V(\mathbf{p}\text{-}\mathbf{p}')$, so for an interaction which falls off sufficiently rapidly the partial amplitudes a_l (where $a_l = (m/4\pi)\tilde{\Gamma}_l$) fall off as $(ap_F)^{2l+1}$ (see Sec. 132 in Ref. 4). This expansion becomes illegitimate, when second-order diagrams are taken into account, however, since the integration over the inner loop of the Green's functions in the zero-sound channel obviously gives rise to a dependence on the angle θ (between the incoming and outgoing momenta) in $\tilde{\Gamma}$.

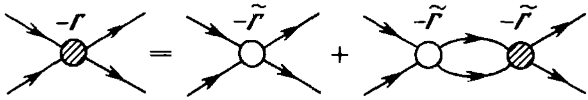

FIG. 1.

Consequently, all the coefficients a_l are real quantities on the order of $(ap_F)^2$. The effective seed vertex $\tilde{\Gamma}$ (in which we retain the spin indices, for accuracy) is given, explicitly by

$$|\mathbf{p}| = |\mathbf{k}| = p_F, \quad \hat{\theta} = \widehat{\mathbf{pk}}$$

$$\tilde{\Gamma}_{\alpha\beta\zeta\eta}(\mathbf{p}, -\mathbf{p}, \mathbf{k}, -\mathbf{k}) = \frac{4\pi}{m}a\left\{\left[1 + \frac{ap_F}{\pi}f(-\cos\theta)\right]\delta_{\alpha\zeta}\right.$$
$$\left.\times\ \delta_{\beta\eta} - \left[1 + \frac{ap_F}{\pi}f(\cos\theta)\right]\delta_{\alpha\eta}\delta_{\beta\zeta}\right\}$$
$$+\ O((ap_F)^3),$$

where

$$f(\cos\theta) = 1 + \frac{\sqrt{2}}{4}\,\frac{1+\cos\theta}{\sqrt{1-\cos\theta}}\ln\frac{\sqrt{2}+\sqrt{1-\cos\theta}}{\sqrt{2}-\sqrt{1-\cos\theta}}\,. \tag{2}$$

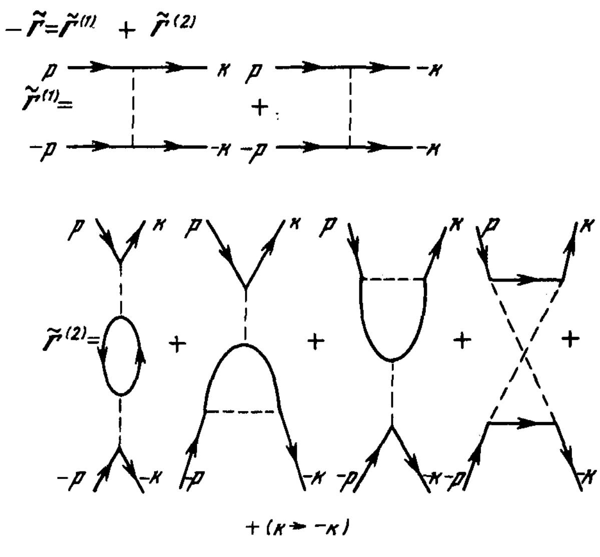

FIG. 2.

Since we are interested in a Fermi gas with repulsion, i.e., the case $a > 0$, we write

$$p_F a_0 = p_F a \left[1 + \frac{2}{3}\,\frac{a p_F}{\pi}(1 + 2\ln 2)\right] > 0\,,$$

and pairing does not occur in the s channel. The following coefficients $a_1, a_2, \ldots$, however, are **negative:**

$$p_F a_1 = \frac{4}{5\pi}(a p_F)^2 (1 - 2\ln 2) < 0$$
$$p_F a_2 = \frac{8}{105\pi}(a p_F)^2 (11\ln 2 - 8) \approx 0{,}1\,(p_F a_1) < 0\,, \qquad (3)$$

etc. Calculations show that the coefficients a_l fall off rapidly with increasing l. The fact that we have $a_1 p_F < 0$ and that $|a_1|$ is at a maximum means that in a low-density Fermi gas with repulsion there is a transition to a superconducting state with an angular momentum $l = 1$ for the Cooper pair at a temperature

$$T_{c1} \sim \epsilon_F \exp\{-\pi/2|a_1|p_F\} \qquad (4)$$

(§54 in Ref. 2). Refining the expression for the argument of the exponential function and calculating the coefficient of the exponential function require calculations for the diagrams of third and fourth orders, which we have not carried out.

III. The physical reason for the appearance of an effective attraction between particles at the Fermi surface is revealed in the large-l limit, which was analyzed in Ref. 3. Specifically, at $l \gg 1$ the dominant component (which falls off as $1/l^4$) of a_l comes from angles θ close to π, where $f(\cos\theta)$ has an $x \ln x$ ($x = \pi - \theta$) singularity, similar to the singularity in the dielectric constant of an electron gas in a metal (the Friedel-Kohn singularity). Accordingly, in a manner similar to that in a metal, the effective potential $\tilde{\Gamma}(r)$ oscillates at large distances in accordance with $(1/r^3)\cos 2k_F r$. Calculations show that at large values of l (for which large distances are important) the oscillations of the interaction are transferred directly to oscillations in the sign of the coefficients in the expansion of $f(\cos\theta)$ [see (2)] in Legendre polynomials: The even harmonics turned out to be negative, and the odd harmonics positive. The circumstance that expression (1) for $\tilde{\Gamma}$ includes $f(-\cos\theta)$, however, means that the signs of all the odd harmonics will change. As a result, regardless of the parity of l, the coefficients a_l will be negative. Kohn and Luttinger's assertion that an effective attraction appears refers only to the case $l \gg 1$. Our calculations show [see (3)] that the effect of the singular part of $f(\cos\theta)$ extends to $l = 1$ and that, furthermore, $|a_1|$ is at a maximum.

IV. Expression (4) should work for weak solutions of ^{3}He in ^{4}He (for which the parameter $a p_F$ is small) in the region of magnetic fields in which a pairing with $S = l = 1$ has been proposed.[5] We note, however, that again in the case of pure ^{3}He a calculation from (4) yields the reasonable result $T_{c1} \sim 10^{-2}\epsilon_F \sim 10^{-2}$ K, although in this case we have $a p_F \sim 2$, and we definitely cannot restrict the analysis to second-order perturbation theory.

These calculations can also be carried out for the electron subsystems in metals, if the interaction between the electrons occurs not through phonons with an energy $\sim\omega_D$ but through intermediate excitations of some sort with an energy $\sim\epsilon_F$.

V. We conclude with a look at the methodological question of calculating the temperature of the transition to the $l=1$ state in the case in which the effective radius of the potential, r_0, is small in comparison with the mean distance between particles, but the s-scattering length satisfies $a\equiv 0$. In other words, we consider the case in which the particle scattering amplitude on the Fermi surface is of the form

$$f(\mathbf{p},\mathbf{p}')p_F = 3\lambda(r_0 p_F)^3\cos\theta + O((r_0 p_F)^5), \qquad \lambda > 0.$$

In this case the problem of determining the transition temperature is solved by the same method as was used by Gor'kov and Melik-Barkhudarov[1] in an analysis of s pairing. All the short-wavelength infinities are eliminated by a renormalization of the coupling of the interaction potential with the p-scattering amplitude. Skipping over the lengthy calculations, we present the final result:

$$T_c \approx 0.10\epsilon_F \exp\left\{-\frac{\pi}{2\lambda(r_0 p_F)^3}\right\}. \tag{5}$$

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